Instabilità elastica: carico di punta

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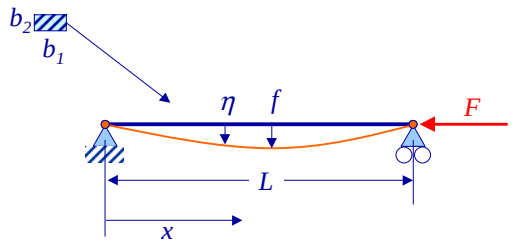

$$\eta'' = -\frac{M}{EJ}$$

$$M = F\eta$$

$$\eta'' = -\frac{F}{EJ}\eta$$

$$a^2 = \frac{F}{EJ} \rightarrow \eta'' = -a^2\eta \rightarrow \eta'' + a^2\eta = 0$$

$$\eta = C_1 \operatorname{sen} ax + C_2 \cos ax$$

$$\eta' = aC_1 \cos ax - aC_2 \operatorname{sen} ax$$

$$\eta'' = -a^2 C_1 \operatorname{sen} ax - a^2 C_2 \operatorname{sen} ax = -a^2\left(C_1 \operatorname{sen} ax + C_2 \operatorname{sen} ax\right) = -a^2\eta$$

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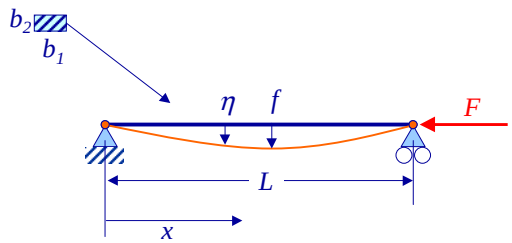


$$\eta'' = -\frac{M}{EJ} \qquad a^2 = \frac{F}{EJ}$$

$$M = F\eta \qquad \eta'' + a^2\eta = 0$$

$$\eta'' = -\frac{F}{EJ}\eta$$

$$\eta = C_1 \operatorname{sen} ax + C_2 \cos ax$$

$$\eta' = aC_1 \cos ax - aC_2 \operatorname{sen} ax$$

$$\eta = C_1 \operatorname{sen} ax$$

per x =0 η =0 $\quad 0 = C_1 \operatorname{sen} 0 + C_2 \cos 0$

$$C_2 = 0$$

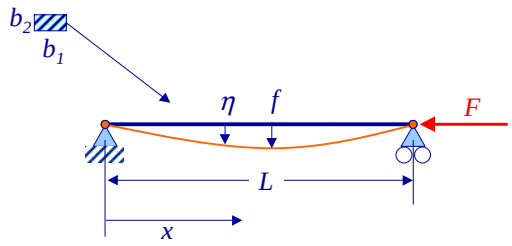
per x =L η =0 $\quad 0 = C_1 \operatorname{sen} aL$

La trave non si inflette $C_1 = 0$ $\qquad \operatorname{sen} aL = 0$

$$aL = n\pi$$

CDM - Instabilità elastica

Instabilità elastica: carico di punta



$$\eta'' = -\frac{M}{EJ} \qquad a^2 = \frac{F}{EJ}$$

$$M = F\eta \qquad \eta'' + a^2\eta = 0$$

$$\eta'' = -\frac{F}{EJ}\eta$$

$$\eta = C_1 \operatorname{sen} ax + C_2 \cos ax$$

$$\eta' = aC_1 \cos ax - aC_2 \operatorname{sen} ax$$

$$\eta = C_1 \operatorname{sen} ax \qquad C_2 = 0$$

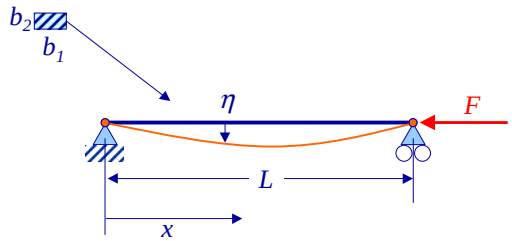
$$aL = n\pi$$

per n =1 $\quad aL = \pi \rightarrow a^2L^2 = \pi^2$

$$a^2 = \frac{\pi^2}{L^2}$$

$$a^2 = \frac{F}{EJ} \qquad \frac{\pi^2}{L^2} = \frac{F}{EJ} \rightarrow F_{cr} = \frac{EJ}{L^2}\pi^2 \rightarrow F_{cr} = \frac{EJ}{L^2}n^2\pi^2$$

Instabilità elastica: carico di punta



$$F_{cr} = \frac{EJ}{L^2}\pi^2$$

Posto:

$L = 2m$
$b_1 = 50mm = 0.05m$
$b_2 = 20mm = 0.02m$

$$F_{cr} = \frac{200E9 \cdot 3.33E-8}{2^2}\pi^2 = 16'449.3 \ N$$

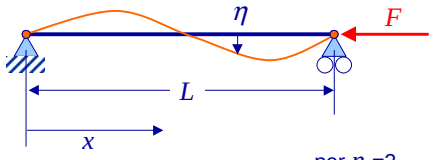
$$A = b_1 \cdot b_2 = 0.05 \cdot 0.02 = 0.001m^2$$

$$J = \frac{b_1 \cdot b_2^{\,3}}{12} = \frac{0.05 \cdot 0.02^3}{12} = 3.333E-8m^4$$

$E = 200GPa$

CDM - Instabilità elastica

Instabilità elastica: carico di punta



$$F_{cr} = \frac{EJ}{L^2}\pi^2$$

per n =2

Posto:

$L = 2m$
$b_1 = 50mm = 0.05m$
$b_2 = 20mm = 0.02m$

$$F_{cr} = \frac{200E9 \cdot 3.33E-8}{2^2}2^2\pi^2 = 65'797.4 \ N$$

$$A = b_1 \cdot b_2 = 0.05 \cdot 0.02 = 0.001m^2$$

$$J = \frac{b_1 \cdot b_2^{\,3}}{12} = \frac{0.05 \cdot 0.02^3}{12} = 3.333E-8m^4$$

$E = 200GPa$