INVARIANTI GEOMETRICI ISTANTANEI

CARATTERISTICHE DELLA MATRICE [u] che è un operatore di derivazione della matrice di rotazione [R]

u_2 è la matrice u al quadrato u^2

etc. etc.

u_inv è l'inversa di u

u_T è la trasposta di u

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Warning, the protected names norm and trace have been redefined and unprotected
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$$u := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$u\_2 := \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$u\_3 := \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$u\_4 := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$u\_5 := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$u\_inv := \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$u\_T := \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

VERIFICA DELLE CARATTERISTICHE DELLA MATRICE [u] ai fini del calcolo delle derivate della matrice di rotazione R
R1 è la derivata della matrice di rotazione calcolata rispetto a phi.
uR è il prodotto matriciale di [u] * [R]
Errore è la differenza tra R1 ed uR

Da notare che per derivare [R] n volte basta semplicemente premoltiplicarla per la matrice [u]
Si vede anche che [u] *[R] = [R]*[u]

$$R := \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}$$

$$R1 := \begin{bmatrix} -\sin(\phi) & -\cos(\phi) \\ \cos(\phi) & -\sin(\phi) \end{bmatrix}$$

$$uR := \begin{bmatrix} -\sin(\phi) & -\cos(\phi) \\ \cos(\phi) & -\sin(\phi) \end{bmatrix}$$

$$R1\_uR\_differenza := \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$R2 := \begin{bmatrix} -\cos(\phi) & \sin(\phi) \\ -\sin(\phi) & -\cos(\phi) \end{bmatrix}$$

$$u\_2R := \begin{bmatrix} -\cos(\phi) & \sin(\phi) \\ -\sin(\phi) & -\cos(\phi) \end{bmatrix}$$

$$Errore2 := \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$Ru := \begin{bmatrix} -\sin(\phi) & -\cos(\phi) \\ \cos(\phi) & -\sin(\phi) \end{bmatrix}$$

$$Ru\_uR\_differenza := \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

USO DELLE FORMULE DI TRASFORMAZIONE

o è l'origine del riferimento mobile (o,x,y)

O è l'origine del riferimento fisso

x,y sono gli assi del riferimento mobile (corrispondono anche alle generiche coordinate del punto generico P nel riferimento mobile)

X,Y sono gli assi del riferimento fisso (corrispondono anche alle generiche coordinate del punto generico P nel riferimento fisso)

phi è l'angolo di posizione del riferimento mobile rispetto a quello fisso

p è il generico punto del piano mobile nel riferimento mobile stesso

Rp è il prodotto [R]*[p]

P è il punto generico p espresso nel riferimento fisso. Notare che P e p sono espressi a partire dalla conoscenza dell'origine o del riferimento mobile (essendo o arbitrario)

In particolare,

[P] = [o] + [R][p] = [o] + [Rp]

$$o := \begin{bmatrix} Xo(\phi) \\ Yo(\phi) \end{bmatrix}$$

$$p := \begin{bmatrix} x \\ y \end{bmatrix}$$

$$Rp := \begin{bmatrix} \cos(\phi)\, x - \sin(\phi)\, y \\ \sin(\phi)\, x + \cos(\phi)\, y \end{bmatrix}$$

$$P := \begin{bmatrix} Xo(\phi) + \cos(\phi)\, x - \sin(\phi)\, y \\ Yo(\phi) + \sin(\phi)\, x + \cos(\phi)\, y \end{bmatrix}$$

CALCOLO DELLE DERIVATE GEOMETRICHE
DELLA POSIZIONE DEL PUNTO GENERICO P
RISPETTO AL PARAMETRO DEL MOTO

Come parametro del moto si prende l'angolo phi di posizione del riferimento mobile

Pdn raprpesenta la drivata n esima del punto P calcolata rispetto all'angolo phi

odn rappresenta la derivata n esima dell'origine o calcolata rispetto all'angolo phi

$$Pd1 := \begin{bmatrix} \left(\frac{d}{d\phi} Xo(\phi)\right) - \sin(\phi)\, x - \cos(\phi)\, y \\ \left(\frac{d}{d\phi} Yo(\phi)\right) + \cos(\phi)\, x - \sin(\phi)\, y \end{bmatrix}$$

$$Pd2 := \begin{bmatrix} \left(\frac{d^2}{d\phi^2} X\alpha(\phi)\right) - \cos(\phi)\, x + \sin(\phi)\, y \\ \left(\frac{d^2}{d\phi^2} Y\alpha(\phi)\right) - \sin(\phi)\, x - \cos(\phi)\, y \end{bmatrix}$$

$$Pd3 := \begin{bmatrix} \left(\frac{d^3}{d\phi^3} X\alpha(\phi)\right) + \sin(\phi)\, x + \cos(\phi)\, y \\ \left(\frac{d^3}{d\phi^3} Y\alpha(\phi)\right) - \cos(\phi)\, x + \sin(\phi)\, y \end{bmatrix}$$

$$Pd4 := \begin{bmatrix} \left(\frac{d^4}{d\phi^4} X\alpha(\phi)\right) + \cos(\phi)\, x - \sin(\phi)\, y \\ \left(\frac{d^4}{d\phi^4} Y\alpha(\phi)\right) + \sin(\phi)\, x + \cos(\phi)\, y \end{bmatrix}$$

$$od1 := \begin{bmatrix} \frac{d}{d\phi} X\alpha(\phi) \\ \frac{d}{d\phi} Y\alpha(\phi) \end{bmatrix}$$

$$od2 := \begin{bmatrix} \frac{d^2}{d\phi^2} X\alpha(\phi) \\ \frac{d^2}{d\phi^2} Y\alpha(\phi) \end{bmatrix}$$

$$od3 := \begin{bmatrix} \frac{d^3}{d\phi^3} Xo(\phi) \\ \frac{d^3}{d\phi^3} Yo(\phi) \end{bmatrix}$$

$$od4 := \begin{bmatrix} \frac{d^4}{d\phi^4} Xo(\phi) \\ \frac{d^4}{d\phi^4} Yo(\phi) \end{bmatrix}$$

DEDUZIONE DELLA POSIZIONE
DEL POLO GEOMETRICO DEL PRIMO ORDINE P1

Sapendo che per il punto [P1] di coordinate [p1] nel riferimento mobile,
devono valere le relazioni

[P] = [o] + [R]*[p]

[Pd1] = [od1] + [u]*[R]*[p1] = [0]

[p1] = - ([u]*[R])^(-1) * [od1] = - R^(-1) * [u]^(-1)   * [od1]

Per seguire i calcoli si consideri che si è posto:
[Rm1] = [R]^-1
[R_1_u_1] = [R]^-1 * [u]^-1
[P1] = [o] + [R] * [p1]

$$Rm1 := \begin{bmatrix} \frac{\cos(\phi)}{\cos(\phi)^2 + \sin(\phi)^2} & \frac{\sin(\phi)}{\cos(\phi)^2 + \sin(\phi)^2} \\ -\frac{\sin(\phi)}{\cos(\phi)^2 + \sin(\phi)^2} & \frac{\cos(\phi)}{\cos(\phi)^2 + \sin(\phi)^2} \end{bmatrix}$$

$$R\_1\_u\_1 := \begin{bmatrix} -\frac{\sin(\phi)}{\cos(\phi)^2 + \sin(\phi)^2} & \frac{\cos(\phi)}{\cos(\phi)^2 + \sin(\phi)^2} \\ -\frac{\cos(\phi)}{\cos(\phi)^2 + \sin(\phi)^2} & -\frac{\sin(\phi)}{\cos(\phi)^2 + \sin(\phi)^2} \end{bmatrix}$$

$$R\_1\_u\_1 := \begin{bmatrix} -\sin(\phi) & \cos(\phi) \\ -\cos(\phi) & -\sin(\phi) \end{bmatrix}$$

$$p1 := \begin{bmatrix} -\sin(\phi)\left(\frac{d}{d\phi} Xo(\phi)\right) + \cos(\phi)\left(\frac{d}{d\phi} Yo(\phi)\right) \\ -\cos(\phi)\left(\frac{d}{d\phi} Xo(\phi)\right) - \sin(\phi)\left(\frac{d}{d\phi} Yo(\phi)\right) \end{bmatrix}$$

$$p1 := \begin{bmatrix} -\sin(\phi)\left(\frac{d}{d\phi} Xo(\phi)\right) + \cos(\phi)\left(\frac{d}{d\phi} Yo(\phi)\right) \\ -\cos(\phi)\left(\frac{d}{d\phi} Xo(\phi)\right) - \sin(\phi)\left(\frac{d}{d\phi} Yo(\phi)\right) \end{bmatrix}$$

$$p1 := \begin{bmatrix} \sin(\phi)\left(\frac{d}{d\phi} Xo(\phi)\right) - \cos(\phi)\left(\frac{d}{d\phi} Yo(\phi)\right) \\ \cos(\phi)\left(\frac{d}{d\phi} Xo(\phi)\right) + \sin(\phi)\left(\frac{d}{d\phi} Yo(\phi)\right) \end{bmatrix}$$

$P1 :=$ [

$$Xa(\phi) + \cos(\phi)\left(\sin(\phi)\left(\frac{d}{d\phi}Xa(\phi)\right) - \cos(\phi)\left(\frac{d}{d\phi}Ya(\phi)\right)\right)$$

$$- \sin(\phi)\left(\cos(\phi)\left(\frac{d}{d\phi}Xa(\phi)\right) + \sin(\phi)\left(\frac{d}{d\phi}Ya(\phi)\right)\right)\Big],$$

$$\Big[$$

$$Ya(\phi) + \sin(\phi)\left(\sin(\phi)\left(\frac{d}{d\phi}Xa(\phi)\right) - \cos(\phi)\left(\frac{d}{d\phi}Ya(\phi)\right)\right)$$

$$+ \cos(\phi)\left(\cos(\phi)\left(\frac{d}{d\phi}Xa(\phi)\right) + \sin(\phi)\left(\frac{d}{d\phi}Ya(\phi)\right)\right)\Big]\Big]$$

$$P1 := \begin{bmatrix} Xa(\phi) - \left(\frac{d}{d\phi}Ya(\phi)\right) \\ Ya(\phi) + \left(\frac{d}{d\phi}Xa(\phi)\right) \end{bmatrix}$$

DEDUZIONE DELLA VELOCITà GEOMETRICA, DELLA ACCELERAZIONE GEOMETRICA, DEL JERK GEOMETRICO E DEL JOPUNCE GEOMETRICO DEL PUNTO P1 PENSATO SEMPRE APPARTENENTE AL SISTEMA MOBILE

Anzitutto occorre notare che se si deriva l'espressione di [P1] non si trova la velocità (nulla) di P1 pensato appartenente al sistema mobile ma si trova invece la velocità con cui il P1 (virtuale) si muove sulla polare fissa al variare di phi.

$$\begin{bmatrix} \left(\frac{d}{d\phi}Xa(\phi)\right) - \left(\frac{d^2}{d\phi^2}Ya(\phi)\right) \\ \left(\frac{d}{d\phi}Ya(\phi)\right) + \left(\frac{d^2}{d\phi^2}Xa(\phi)\right) \end{bmatrix}$$

Occorre invece considerare le espressioni trovate delle velocità geometriche, accelerazione etc. etc. ed APPLICARLE AL PUNTO [p1] DEL PIANO MOBILE.

Si ricorda che nel linguaggio di programmazione usato i vettori sono visti come matrici aventi una sola colonna. Quindi il secondo indice deve sempre essere = 1.

Il vettore [P1dn] raprpesenta la derivata n esima del vettore [P1] rappresentante della posizione del centro di rotazione istantaneo geometrico nel riferimento fisso, calcolata rispetto al parametro phi. Ovviamente la derivata prima è nulla poicè la velocità geometrica deve essere nulla per il polo geometrico del primo ordine.

$$X\alpha(\phi) - \left(\frac{d}{d\phi} Y\alpha(\phi)\right)$$

$$Y\alpha(\phi) + \left(\frac{d}{d\phi} X\alpha(\phi)\right)$$

$$P1d1 := \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$P1d2 := \begin{bmatrix} \left(\frac{d}{d\phi} Y\alpha(\phi)\right) + \left(\frac{d^2}{d\phi^2} X\alpha(\phi)\right) \\ \left(\frac{d^2}{d\phi^2} Y\alpha(\phi)\right) - \left(\frac{d}{d\phi} X\alpha(\phi)\right) \end{bmatrix}$$

$$P1d3 := \begin{bmatrix} \left(\frac{d^3}{d\phi^3} X\alpha(\phi)\right) + \left(\frac{d}{d\phi} X\alpha(\phi)\right) \\ \left(\frac{d^3}{d\phi^3} Y\alpha(\phi)\right) + \left(\frac{d}{d\phi} Y\alpha(\phi)\right) \end{bmatrix}$$

$$P1d4 := \begin{bmatrix} \left(\frac{d^4}{d\phi^4} X\alpha(\phi)\right) - \left(\frac{d}{d\phi} Y\alpha(\phi)\right) \\ \left(\frac{d^4}{d\phi^4} Y\alpha(\phi)\right) + \left(\frac{d}{d\phi} X\alpha(\phi)\right) \end{bmatrix}$$

CALCOLO DEGLI INVARIANTI GEOMETRICI

Per calcolare gli invarianti di ordine superiore dapprima calcoliamo i versori N e T, rappresentativi dei versori principali del RIFERIMENTO CANONICO. I versori T ed N devono essere espressi nel riferimento fisso X,Y.

Per contro, saranno chiamati e1 ed e2 i versori associati agli assi fissi X ed Y.

Operativamente T ed N sono rappresentati da un vettore colonna a 2 dimenisoni i cui elementi sono le componenti lungo X ed Y.

Si comincia con il determinare il versore N. Infatti, l'accelerazione del centro delle velocità geometrico è diretta proprio secondo l'asse N. Quindi, basta imporre che tale vettore, espresso secondo e1 ed e2, sia uguale al vettore espresso in N e T essendo nulla la componente lungo T. Deve risultare

[N] = P1d2x / b2 [e1] + P1d2y / b2 [e2]

Il versore [T] si trova ruotando [N] di 90 °.

$$N := \begin{bmatrix} \dfrac{\left(\dfrac{d}{d\phi} Ya(\phi)\right) + \left(\dfrac{d^2}{d\phi^2} Xa(\phi)\right)}{b_2} \\ -\dfrac{\left(\dfrac{d}{d\phi} Xa(\phi)\right) - \left(\dfrac{d^2}{d\phi^2} Ya(\phi)\right)}{b_2} \end{bmatrix}$$

$$T := \begin{bmatrix} -\dfrac{\left(\dfrac{d}{d\phi} Xa(\phi)\right) - \left(\dfrac{d^2}{d\phi^2} Ya(\phi)\right)}{b_2} \\ -\dfrac{\left(\dfrac{d}{d\phi} Ya(\phi)\right) + \left(\dfrac{d^2}{d\phi^2} Xa(\phi)\right)}{b_2} \end{bmatrix}$$

METTENDOSI INVECE IN UN SISTEMA RIFERITO AI VERSORI N E T, I VERSORI e1 ed e2 HANNO LE SEGUENTI COORDINATE

$$e1 := \begin{bmatrix} -\dfrac{\left(\dfrac{d}{d\phi} Xa(\phi)\right) - \left(\dfrac{d^2}{d\phi^2} Ya(\phi)\right)}{b_2} \\ \dfrac{\left(\dfrac{d}{d\phi} Ya(\phi)\right) + \left(\dfrac{d^2}{d\phi^2} Xa(\phi)\right)}{b_2} \end{bmatrix}$$

$$e2 := \left[\begin{array}{c} -\dfrac{\left(\dfrac{d}{d\phi} Ya(\phi)\right) + \left(\dfrac{d^2}{d\phi^2} Xa(\phi)\right)}{b_2} \\ -\dfrac{\left(\dfrac{d}{d\phi} Xa(\phi)\right) - \left(\dfrac{d^2}{d\phi^2} Ya(\phi)\right)}{b_2} \end{array}\right]$$

Gli invarianti del terzo e del quarto ordine sono ora memorizzati nei vettori [AB3] ed [AB4] essendo:

[AB3] = P1d3[1,1] * [e1] + P1d3[2,1] * [e2]
[AB4] = P1d4[1,1] * [e1] + P1d4[2,1] * [e2]

In questi vettori si opera la sostituzione dei versori [e1] ed [e2], ottenuti nel passaggio precedente, ottenendo le espressioni nelle coponenti T ed N.

$$AB3 := \left[\left[-\frac{\left(\left(\dfrac{d^3}{d\phi^3} Xa(\phi)\right) + \left(\dfrac{d}{d\phi} Xa(\phi)\right)\right)\left(\left(\dfrac{d}{d\phi} Xa(\phi)\right) - \left(\dfrac{d^2}{d\phi^2} Ya(\phi)\right)\right)}{b_2} \right.\right.$$

$$-\frac{\left(\left(\frac{d^3}{d\phi^3}Ya(\phi)\right)+\left(\frac{d}{d\phi}Ya(\phi)\right)\right)\left(\left(\frac{d}{d\phi}Ya(\phi)\right)+\left(\frac{d^2}{d\phi^2}Xa(\phi)\right)\right)}{b_2}\Bigg],$$

$$\Bigg[\frac{\left(\left(\frac{d^3}{d\phi^3}Xa(\phi)\right)+\left(\frac{d}{d\phi}Xa(\phi)\right)\right)\left(\left(\frac{d}{d\phi}Ya(\phi)\right)+\left(\frac{d^2}{d\phi^2}Xa(\phi)\right)\right)}{b_2}$$

$$-\frac{\left(\left(\frac{d^3}{d\phi^3}Ya(\phi)\right)+\left(\frac{d}{d\phi}Ya(\phi)\right)\right)\left(\left(\frac{d}{d\phi}Xa(\phi)\right)-\left(\frac{d^2}{d\phi^2}Ya(\phi)\right)\right)}{b_2}\Bigg]\Bigg]$$

$$AB4 := \Bigg[\Bigg[-\frac{\left(\left(\frac{d^4}{d\phi^4}Xa(\phi)\right)-\left(\frac{d}{d\phi}Ya(\phi)\right)\right)\left(\left(\frac{d}{d\phi}Xa(\phi)\right)-\left(\frac{d^2}{d\phi^2}Ya(\phi)\right)\right)}{b_2}$$

$$- \frac{\left(\left(\frac{d^4}{d\phi^4} Ya(\phi)\right) + \left(\frac{d}{d\phi} Xa(\phi)\right)\right)\left(\left(\frac{d}{d\phi} Ya(\phi)\right) + \left(\frac{d^2}{d\phi^2} Xa(\phi)\right)\right)}{b_2}\Bigg],$$

$$\Bigg[\frac{\left(\left(\frac{d^4}{d\phi^4} Xa(\phi)\right) - \left(\frac{d}{d\phi} Ya(\phi)\right)\right)\left(\left(\frac{d}{d\phi} Ya(\phi)\right) + \left(\frac{d^2}{d\phi^2} Xa(\phi)\right)\right)}{b_2}$$

$$- \frac{\left(\left(\frac{d^4}{d\phi^4} Ya(\phi)\right) + \left(\frac{d}{d\phi} Xa(\phi)\right)\right)\left(\left(\frac{d}{d\phi} Xa(\phi)\right) - \left(\frac{d^2}{d\phi^2} Ya(\phi)\right)\right)}{b_2}\Bigg]\Bigg]$$

DEDUZIONE DELLA CIRCONFERENZA DEI FLESSI,
DELLA CUBICA DI CURVATURA STAZIONARIA
E DELLA CUBICA DELLA DERIVATA SECONDA DELLA CURVATURA STAZIONARIA

Per dedurre la circonferenza dei flessi occorre ancora imporre che la velocità sia parallele alla accelerazione (sempre da intendere come derivate geometriche). Si impone quindi che il loro prodotto vettoriale sia nullo.

$$ku := \frac{\Xi1(\chi)\,Y2(\chi) - \Xi2(\chi)\,Y1(\chi)}{(\Xi1(\chi)^2 + Y1(\chi)^2)^{(3/2)}}$$

$$kud1 := \frac{\Xi1(\chi)\,Y3(\chi) - \Xi3(\chi)\,Y1(\chi)}{(\Xi1(\chi)^2 + Y1(\chi)^2)^{(3/2)}} - \frac{3\,(\Xi1(\chi)\,Y2(\chi) - \Xi2(\chi)\,Y1(\chi))\,(2\,\Xi1(\chi)\,\Xi2(\chi) + 2\,Y1(\chi)\,Y2(\chi))}{2\,(\Xi1(\chi)^2 + Y1(\chi)^2)^{(5/2)}}$$

$$kud2 := (2\,\Xi1(\chi)\,\Xi2(\chi) + 2\,Y1(\chi)\,Y2(\chi))\,\Xi1(\chi)\,Y3(\chi) + (\Xi1(\chi)^2 + Y1(\chi)^2)\,\Xi2(\chi)\,Y3(\chi)$$

$$+ (\Xi1(\chi)^2 + Y1(\chi)^2)\,\Xi1(\chi)\,Y4(\chi) - (2\,\Xi1(\chi)\,\Xi2(\chi) + 2\,Y1(\chi)\,Y2(\chi))\,\Xi3(\chi)\,Y1(\chi) - (\Xi1(\chi)^2 + Y1(\chi)^2)\,\Xi4(\chi)\,Y1(\chi) - (\Xi1(\chi)^2 + Y1(\chi)^2)\,\Xi3(\chi)\,Y2(\chi) - 3\,\Xi1(\chi)\,Y2(\chi)\,\Xi2(\chi)^2 - 3\,\Xi1(\chi)^2\,Y3(\chi)\,\Xi2(\chi) - 3\,\Xi1(\chi)^2\,Y2(\chi)\,\Xi3(\chi) + 3\,\Xi2(\chi)\,Y2(\chi)^2\,Y1(\chi) - 6\,\Xi1(\chi)\,Y2(\chi)\,Y1(\chi)\,Y3(\chi) - 3\,\Xi1(\chi)\,Y2(\chi)^3 + 6\,\Xi2(\chi)\,Y1(\chi)\,\Xi1(\chi)\,\Xi3(\chi) + 3\,\Xi2(\chi)^3\,Y1(\chi) + 3\,\Xi3(\chi)\,Y1(\chi)^2\,Y2(\chi) + 3\,\Xi2(\chi)\,Y1(\chi)^2\,Y3(\chi)$$

$$fless := \Xi1(\chi)\,Y2(\chi) - \Xi2(\chi)\,Y1(\chi)$$

$$fless := -y\,b2 + y^2 + x^2$$

$$cucusta = (\Xi1(\chi)^2 + Y1(\chi)^2)\,\Xi1(\chi)\,Y3(\chi) - (\Xi1(\chi)^2 + Y1(\chi)^2)\,\Xi3(\chi)\,Y1(\chi) - 3\,\Xi1(\chi)^2\,Y2(\chi)\,\Xi2(\chi) - 3\,\Xi1(\chi)\,Y2(\chi)^2\,Y1(\chi) + 3\,\Xi2(\chi)^2\,Y1(\chi)\,\Xi1(\chi) + 3\,\Xi2(\chi)\,Y1(\chi)^2\,Y2(\chi)$$

$$cucusta = -y^3\,b3 - y\,x^2\,b3 - x\,y^2\,a3 - x^3\,a3 - 3\,y^2\,x\,b2 + 3\,y\,x\,b2^2 - 3\,x^3\,b2$$

$$cucu2 := -4\,\Xi1(\chi)\,Y2(\chi)\,Y1(\chi)\,Y3(\chi) + 4\,\Xi2(\chi)\,Y1(\chi)^2\,Y3(\chi) + \Xi1(\chi)^3\,Y4(\chi) + \Xi1(\chi)\,Y4(\chi)\,Y1(\chi)^2 + 4\,\Xi2(\chi)\,Y1(\chi)\,\Xi1(\chi)\,\Xi3(\chi) - \Xi4(\chi)\,Y1(\chi)\,\Xi1(\chi)^2 - \Xi4(\chi)\,Y1(\chi)^3 - 4\,\Xi1(\chi)^2\,Y2(\chi)\,\Xi3(\chi) - 3\,\Xi1(\chi)\,Y2(\chi)\,\Xi2(\chi)^2 + 3\,\Xi2(\chi)\,Y2(\chi)^2\,Y1(\chi) - 3\,\Xi1(\chi)\,Y2(\chi)^3 + 3\,\Xi2(\chi)^3\,Y1(\chi)$$

$$cucu2 := -4\,y^2\,b2\,a3 + 4\,y\,x\,b2\,b3 + 5\,y\,x^2\,b2 - 4\,y^2\,x\,b3 - x^2\,y\,b4 + 4\,x^2\,y\,a3 - y^2\,x\,a4 - 4\,x^3\,b3 - y^3\,b4 - x^3\,a4 + 5\,y^3\,b2 + 4\,y^3\,a3 - 3\,x^2\,b2^2 + 3\,y\,b2^3 - 9\,y^2\,b2^2$$

Inoltre, occorre imporre che l'angolo phi sia nullo in quanto vogliamo esprimere questo luogo di punti nel riferimento mobile avente assi coincidenti con quello canonico (ritenuto fisso).

$$kud1s := -y^3\,b3 - y\,x^2\,b3 - x\,y^2\,a3 - x^3\,a3 - 3\,y^2\,x\,b2 + 3\,y\,x\,b2^2 - 3\,x^3\,b2$$

$$flessi := -y\,b2 + y^2 + x^2 = 0$$

$$cubica\_cu\_sta := -y^3\,b3 - y\,x^2\,b3 - x\,y^2\,a3 - x^3\,a3 - 3\,y^2\,x\,b2 + 3\,y\,x\,b2^2 - 3\,x^3\,b2 = 0$$

$$cu\_cu\_2 := -4\,y^2\,b2\,a3 + 4\,y\,x\,b2\,b3 + 5\,y\,x^2\,b2 - 4\,y^2\,x\,b3 - x^2\,y\,b4 + 4\,x^2\,y\,a3 - y^2\,x\,a4 - 4\,x^3\,b3 - y^3\,b4 - x^3\,a4 + 5\,y^3\,b2 + 4\,y^3\,a3 - 3\,x^2\,b2^2 + 3\,y\,b2^3 - 9\,y^2\,b2^2 = 0$$

Calcolo del primo semplice invariante b2

$$b_2 := \mathrm{sqrt}\left(\left(\frac{d^2}{d\phi^2} X\alpha(\phi)\right)^2 + 2 \left(\frac{d^2}{d\phi^2} X\alpha(\phi)\right) \left(\frac{d}{d\phi} Y\alpha(\phi)\right) + \left(\frac{d}{d\phi} Y\alpha(\phi)\right)^2 + \left(\frac{d^2}{d\phi^2} Y\alpha(\phi)\right)^2 - 2 \left(\frac{d^2}{d\phi^2} Y\alpha(\phi)\right) \left(\frac{d}{d\phi} X\alpha(\phi)\right) + \left(\frac{d}{d\phi} X\alpha(\phi)\right)^2 \right)$$

$$a_3 := - \left(\left(\frac{d^3}{d\phi^3} X\alpha(\phi)\right) \left(\frac{d}{d\phi} X\alpha(\phi)\right) - \left(\frac{d^3}{d\phi^3} X\alpha(\phi)\right) \left(\frac{d^2}{d\phi^2} Y\alpha(\phi)\right) + \left(\frac{d}{d\phi} X\alpha(\phi)\right)^2 - \left(\frac{d^2}{d\phi^2} Y\alpha(\phi)\right) \left(\frac{d}{d\phi} X\alpha(\phi)\right) + \left(\frac{d^3}{d\phi^3} Y\alpha(\phi)\right) \left(\frac{d^2}{d\phi^2} X\alpha(\phi)\right) + \left(\frac{d^3}{d\phi^3} Y\alpha(\phi)\right) \left(\frac{d}{d\phi} Y\alpha(\phi)\right) + \left(\frac{d^2}{d\phi^2} X\alpha(\phi)\right) \left(\frac{d}{d\phi} Y\alpha(\phi)\right) + \left(\frac{d}{d\phi} Y\alpha(\phi)\right)^2 \right) \Big/ \mathrm{sqrt}\left(\left(\frac{d^2}{d\phi^2} X\alpha(\phi)\right)^2 + 2 \left(\frac{d^2}{d\phi^2} X\alpha(\phi)\right) \left(\frac{d}{d\phi} Y\alpha(\phi)\right) + \left(\frac{d}{d\phi} Y\alpha(\phi)\right)^2 + \left(\frac{d^2}{d\phi^2} Y\alpha(\phi)\right)^2 - 2 \left(\frac{d^2}{d\phi^2} Y\alpha(\phi)\right) \left(\frac{d}{d\phi} X\alpha(\phi)\right) + \left(\frac{d}{d\phi} X\alpha(\phi)\right)^2 \right)$$

$$b_3 := \left(\left(\frac{d^3}{d\phi^3} X\alpha(\phi)\right) \left(\frac{d^2}{d\phi^2} X\alpha(\phi)\right) + \left(\frac{d^3}{d\phi^3} X\alpha(\phi)\right) \left(\frac{d}{d\phi} Y\alpha(\phi)\right) + \left(\frac{d}{d\phi} X\alpha(\phi)\right) \left(\frac{d^2}{d\phi^2} X\alpha(\phi)\right)\right.$$

$$
- \left(\frac{d^{3}}{d\phi^{3}} Ya(\phi)\right) \left(\frac{d}{d\phi} Xa(\phi)\right) + \left(\frac{d^{3}}{d\phi^{3}} Ya(\phi)\right) \left(\frac{d^{2}}{d\phi^{2}} Ya(\phi)\right) + \left(\frac{d}{d\phi} Ya(\phi)\right) \left(\frac{d^{2}}{d\phi^{2}} Ya(\phi)\right) \Bigg) \Bigg) \Bigg/ \mathrm{sqrt}\Bigg(\left(\frac{d^{2}}{d\phi^{2}} Xa(\phi)\right)^{2} + 2 \left(\frac{d^{2}}{d\phi^{2}} Xa(\phi)\right) \left(\frac{d}{d\phi} Ya(\phi)\right) + \left(\frac{d}{d\phi} Ya(\phi)\right)^{2} + \left(\frac{d^{2}}{d\phi^{2}} Ya(\phi)\right)^{2} - 2 \left(\frac{d^{2}}{d\phi^{2}} Ya(\phi)\right) \left(\frac{d}{d\phi} Xa(\phi)\right) + \left(\frac{d}{d\phi} Xa(\phi)\right)^{2} \Bigg)
$$

$$
a_{4} := - \Bigg(\left(\frac{d^{4}}{d\phi^{4}} Xa(\phi)\right) \left(\frac{d}{d\phi} Xa(\phi)\right) - \left(\frac{d^{4}}{d\phi^{4}} Xa(\phi)\right) \left(\frac{d^{2}}{d\phi^{2}} Ya(\phi)\right) + \left(\frac{d}{d\phi} Ya(\phi)\right) \left(\frac{d^{2}}{d\phi^{2}} Ya(\phi)\right) + \left(\frac{d^{4}}{d\phi^{4}} Ya(\phi)\right) \left(\frac{d^{2}}{d\phi^{2}} Xa(\phi)\right) + \left(\frac{d^{4}}{d\phi^{4}} Ya(\phi)\right) \left(\frac{d}{d\phi} Ya(\phi)\right) + \left(\frac{d}{d\phi} Xa(\phi)\right) \left(\frac{d^{2}}{d\phi^{2}} Xa(\phi)\right) \Bigg) \Bigg/ \mathrm{sqrt}\Bigg(\left(\frac{d^{2}}{d\phi^{2}} Xa(\phi)\right)^{2} + 2 \left(\frac{d^{2}}{d\phi^{2}} Xa(\phi)\right) \left(\frac{d}{d\phi} Ya(\phi)\right) + \left(\frac{d}{d\phi} Ya(\phi)\right)^{2} + \left(\frac{d^{2}}{d\phi^{2}} Ya(\phi)\right)^{2} - 2 \left(\frac{d^{2}}{d\phi^{2}} Ya(\phi)\right) \left(\frac{d}{d\phi} Xa(\phi)\right) + \left(\frac{d}{d\phi} Xa(\phi)\right)^{2} \Bigg)
$$

$$b_4 := -\left(-\left(\frac{d^4}{d\phi^4}Xo(\phi)\right)\left(\frac{d^2}{d\phi^2}Xo(\phi)\right) - \left(\frac{d^4}{d\phi^4}Xo(\phi)\right)\left(\frac{d}{d\phi}Yo(\phi)\right) + \left(\frac{d^2}{d\phi^2}Xo(\phi)\right)\left(\frac{d}{d\phi}Yo(\phi)\right) + \left(\frac{d}{d\phi}Yo(\phi)\right)^2 + \left(\frac{d^4}{d\phi^4}Yo(\phi)\right)\left(\frac{d}{d\phi}Xo(\phi)\right) - \left(\frac{d^4}{d\phi^4}Yo(\phi)\right)\left(\frac{d^2}{d\phi^2}Yo(\phi)\right) + \left(\frac{d}{d\phi}Xo(\phi)\right)^2 - \left(\frac{d^2}{d\phi^2}Yo(\phi)\right)\left(\frac{d}{d\phi}Xo(\phi)\right)\right) \Big/ \mathrm{sqrt}\left(\left(\frac{d^2}{d\phi^2}Xo(\phi)\right)^2 + 2\left(\frac{d^2}{d\phi^2}Xo(\phi)\right)\left(\frac{d}{d\phi}Yo(\phi)\right) + \left(\frac{d}{d\phi}Yo(\phi)\right)^2 + \left(\frac{d^2}{d\phi^2}Yo(\phi)\right)^2 - 2\left(\frac{d^2}{d\phi^2}Yo(\phi)\right)\left(\frac{d}{d\phi}Xo(\phi)\right) + \left(\frac{d}{d\phi}Xo(\phi)\right)^2\right)$$

FASE DI ASSEGNAZIONE DEI PARAMETRI NUMERICI
CASO DEL MANOVELLISMO ORDINARIO

Assegnazione della funzione Xo(phi) ed Yo(phi)

$$Xo(\phi) := l1\sqrt{1 - \frac{l2^2 \sin(\phi)^2}{l1^2}}$$

$$Yo(\phi) := -l2 \sin(\phi)$$

ASSEGNAZIONE DEI PARAMETRI INPUT E CALCOLO DEGLI INVARIANTI GEOMETRICI

$$l1 := 1$$

$$l2 := 2.5$$

$$psi\_gradi := 39$$

$$\psi := 0.6806784085$$

$$sen\_phi := -.2517281565$$

$$cos\_phi := 0.9677979826$$

$$assign$$

$$b_2 := 14.61314116$$

$$a_3 := -14.98592242$$

$$b_3 := -83.26076534$$

$$a_4 := 242.6225066$$

$$b_4 := 1333.474502$$

$$P1_{1,\,1} := l1\sqrt{1. - \frac{0.06336706477\, l2^2}{l1^2}} + 0.9677979826\, l2$$

$$P1_{2,\,1} := 0.2517281565\, l2 + \frac{0.2436220020\, l2^2}{l1\sqrt{1. - \frac{0.06336706477\, l2^2}{l1^2}}}$$

$$T_{1,\,1} := -0.01722614965\, l2 - \frac{0.01667143288\, l2^2}{l1\sqrt{1. - \frac{0.06336706477\, l2^2}{l1^2}}}$$

$$T_{2,\,1} := 0.06622792267\, l2 + \frac{0.004061527854\, l2^4}{l1^3\left(1. - \frac{0.06336706477\, l2^2}{l1^2}\right)^{(3/2)}} + \frac{0.05975894306\, l2^2}{l1\sqrt{1. - \frac{0.06336706477\, l2^2}{l1^2}}}$$

Assegnazione della circonferenza dei flessi e della cubica di curvatura stazionaria

$$FLE := -14.61314116\, \eta + \eta^2 + \xi^2 = 0$$

$$CUCUSTA := 83.26076534\, \eta^3 + 83.26076534\, \eta\, \xi^2 - 28.85350106\, \eta^2\, \xi - 28.85350106\, \xi^3 + 640.6316838\, \eta\, \xi = 0$$

$$cucupar := -28.85350106\, h^3 \cos(\zeta) + 640.6316838\, h^2 \sin(\zeta)\cos(\zeta) + 83.26076534 \sin(\zeta)\, h^3$$

$$cucupar := -28.85350106\, h \cos(\zeta) + 640.6316838 \sin(\zeta)\cos(\zeta) + 83.26076534\, h \sin(\zeta)$$

$$h(\zeta) := -\frac{3.203158419\ 10^{10} \sin(\zeta)\cos(\zeta)}{-1.442675053\ 10^{9} \cos(\zeta) + 4.163038267\ 10^{9} \sin(\zeta)}$$

$$CUCU\_2D := -1045.929452\,\eta^2 - 4866.805268\,\eta\,\xi - 1320.352486\,\eta\,\xi^2 + 90.4205548\,\eta^2\,\xi + 90.4205548\,\xi^3 - 1320.352486\,\eta^3 - 640.6316838\,\xi^2 + 9361.641225\,\eta = 0$$

$$cucu\_2D\_par := 405.2977682\,h^2 \cos(\zeta)^2 - 1045.929452\,h^2 + 90.42055480\,h^3 \cos(\zeta) - 4866.805268\,h^2 \sin(\zeta)\cos(\zeta) + 9361.641225\,h \sin(\zeta) - 1320.352486 \sin(\zeta)\,h^3$$

$$cucu\_2D\_par := -\frac{1}{h^3}\Big(2.000000000\ 10^{-7}\big(-2.026488841\ 10^{9}\,h\cos(\zeta)^2 + 5.229647260\ 10^{9}\,h - 4.52102774\ 10^{8}\,h^2\cos(\zeta) + 2.433402634\ 10^{10}\,h\sin(\zeta)\cos(\zeta) - 4.680820612\ 10^{10}\sin(\zeta) + 6.601762430\ 10^{9}\sin(\zeta)\,h^2\big)\Big)$$

$$h2(\zeta) := \frac{1}{2.189383205\ 10^{19}\sin(\zeta)^2 - 1.021984591\ 10^{17}}\Big(0.5000000000\big(-1.607398586\ 10^{19}\sin(\zeta) - 8.078182104\ 10^{19}\sin(\zeta)^2\cos(\zeta) - 1.188458542\ 10^{18}\sin(\zeta)^3 - 7.240784034\ 10^{17}\cos(\zeta) + \mathrm{sqrt}\big(1.952109691\ 10^{38}\sin(\zeta)^2 + 3.525332568\ 10^{39}\sin(\zeta)^3\cos(\zeta) + 1.356932409\ 10^{40}\sin(\zeta)^4 + 1.895217996\ 10^{37}\sin(\zeta)\cos(\zeta) + 6.525702611\ 10^{39}\sin(\zeta)^4\cos(\zeta)^2 + 1.920116906\ 10^{38}\sin(\zeta)^5\cos(\zeta) + 1.169847440\ 10^{38}\sin(\zeta)^2\cos(\zeta)^2 + 1.412433707\ 10^{36}\sin(\zeta)^6 + 5.242895343\ 10^{35}\cos(\zeta)^2\big)\big)\Big),$$

$$\frac{1}{2.189383205\ 10^{19}\sin(\zeta)^2 - 1.021984591\ 10^{17}}\Big(0.5000000000\big(-1.607398586\ 10^{19}\sin(\zeta) - 8.078182104\ 10^{19}\sin(\zeta)^2\cos(\zeta) - 1.188458542\ 10^{18}\sin(\zeta)^3 - 7.240784034\ 10^{17}\cos(\zeta) - 1.\ \mathrm{sqrt}\big(1.952109691\ 10^{38}\sin(\zeta)^2 + 3.525332568\ 10^{39}\sin(\zeta)^3\cos(\zeta) + 1.356932409\ 10^{40}\sin(\zeta)^4 + 1.895217996\ 10^{37}\sin(\zeta)\cos(\zeta) + 6.525702611\ 10^{39}\sin(\zeta)^4\cos(\zeta)^2 + 1.920116906\ 10^{38}\sin(\zeta)^5\cos(\zeta) + 1.169847440\ 10^{38}\sin(\zeta)^2\cos(\zeta)^2 + 1.412433707\ 10^{36}\sin(\zeta)^6$$

$$+ 5.242895343\ 10^{35} \cos(\zeta)^2 \Big) \Big) \Big)$$

A QUESTO PUNTO SI OPERA UN CAMBIO DI COORDINATE: dapprima una rotazione attorno a P1, in modo che il riferimento abbia assi paralleli a quello fisso. Successivamente una traslazione da P1 all'origine del sistema fisso.

In tal modo le due equazioni risultano descritte nel riferimento fisso.

$$flexi := -14.61314116\,\eta + \eta^2 + \xi^2 = 0$$

$$FLE := 14.38204094\,x - 52.67501239 + 2.588588776\,y + (-0.9841854520\,x + 3.604633104 - 0.1771411600\,y)^2 + (-0.1771411600\,x - 1.981394735 + 0.9841854520\,y)^2 = 0.$$

$$cuxi := 83.26076534\,\eta^3 + 83.26076534\,\eta\,\xi^2 - 28.85350106\,\eta^2\,\xi - 28.85350106\,\xi^3 + 640.6316838\,\eta\,\xi = 0$$

$$CUCUSTA := 83.26076534\,(-0.9841854520\,x + 3.604633104 - 0.1771411600\,y)^3 + 83.26076534\,(-0.9841854520\,x + 3.604633104 - 0.1771411600\,y)\,(-0.1771411600\,x - 1.981394735 + 0.9841854520\,y)^2 - 28.85350106\,(-0.9841854520\,x + 3.604633104 - 0.1771411600\,y)^2\,(-0.1771411600\,x - 1.981394735 + 0.9841854520\,y) - 28.85350106\,(-0.1771411600\,x - 1.981394735 + 0.9841854520\,y)^3 + 640.6316838\,(-0.9841854520\,x + 3.604633104 - 0.1771411600\,y)\,(-0.1771411600\,x - 1.981394735 + 0.9841854520\,y) = 0.$$

$$CUCUSTA := -2744.036426\,x - 82.20445111\,y + 1469.657450 - 43.14610430\,x^2\,y + 73.19579374\,x\,y - 76.83289090\,x\,y^2 + 468.9821575\,y^2 + 960.1965790\,x^2 - 76.83289090\,x^3 - 43.14610430\,y^3 = 0.$$

$$cuxi2 := -1045.929452\,\eta^2 - 4866.805268\,\eta\,\xi - 1320.352486\,\eta\,\xi^2 + 90.4205548\,\eta^2\,\xi + 90.4205548\,\xi^3 - 1320.352486\,\eta^3 - 640.6316838\,\xi^2 + 9361.641225\,\eta = 0$$

$$CUCU\_2D := -1045.929452\,(-0.9841854520\,x + 3.604633104 - 0.1771411600\,y)^2 - 4866.805268\,(-0.9841854520\,x + 3.604633104 - 0.1771411600\,y)\,(-0.1771411600\,x - 1.981394735 + 0.9841854520\,y) - 1320.352486\,(-0.9841854520\,x + 3.604633104 - 0.1771411600\,y)\,(-0.1771411600\,x - 1.981394735 + 0.9841854520\,y)^2 + 90.4205548\,(-0.9841854520\,x + 3.604633104 - 0.1771411600\,y)^2\,(-0.1771411600\,x - 1.981394735 + 0.9841854520\,y) + 90.4205548\,(-0.1771411600\,x - 1.981394735 + 0.9841854520\,y)^3 - 1320.352486\,(-0.9841854520\,x + 3.604633104 - 0.1771411600\,y)^3 - 640.6316838\,(-0.1771411600\,x - 1.981394735 + 0.9841854520\,y)^2 - 9213.591100\,x + 33745.28187 - 1658.331986\,y = 0.$$

$$CUCU\_2D := 44663.54144\,x + 14232.74446\,y + 322.8793638\,x^2\,y - 4288.875252\,x\,y + 1283.454499\,x\,y^2$$

$- 6415.020747\, y^2 - 15025.72064\, x^2 + 1283.454499\, x^3 - 31156.99726 + 322.8793638\, y^3 = 0.$

GRAFICO DEL MANOVELLISMO CON LA CIRCONFERENZA DEI FLESSI

BOTT_MAN_x := 0.7771459613

BOTT_MAN_y := 0.6293203912

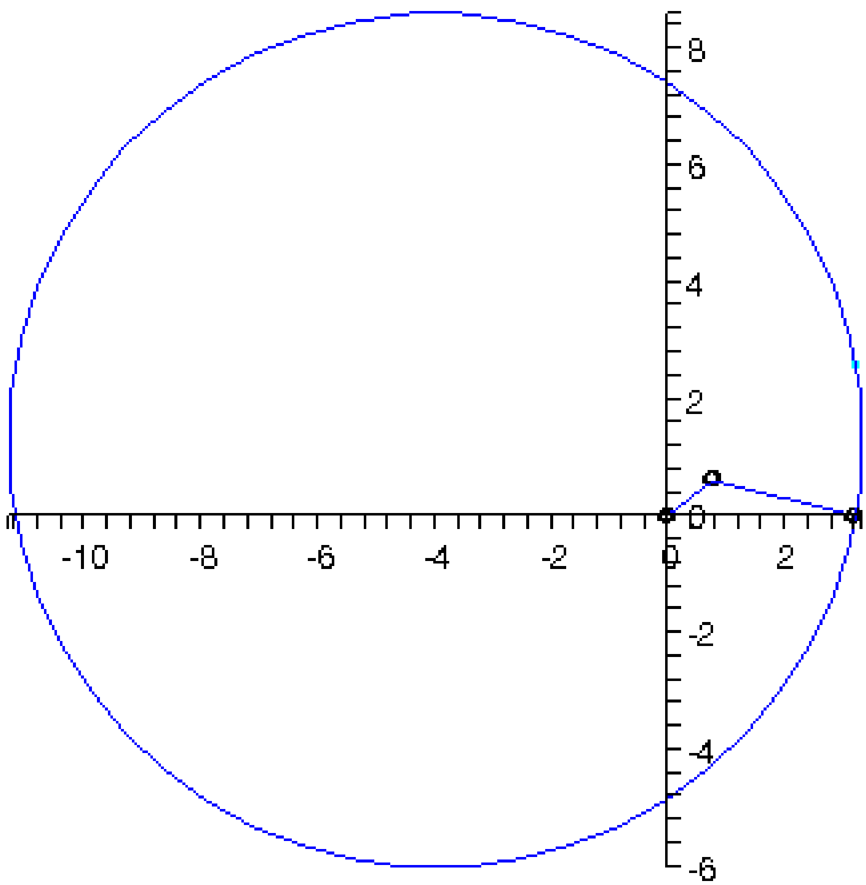
SPINA_x := 3.196640917

SPINA_y := 0.

rag_cer := 0.1111111111

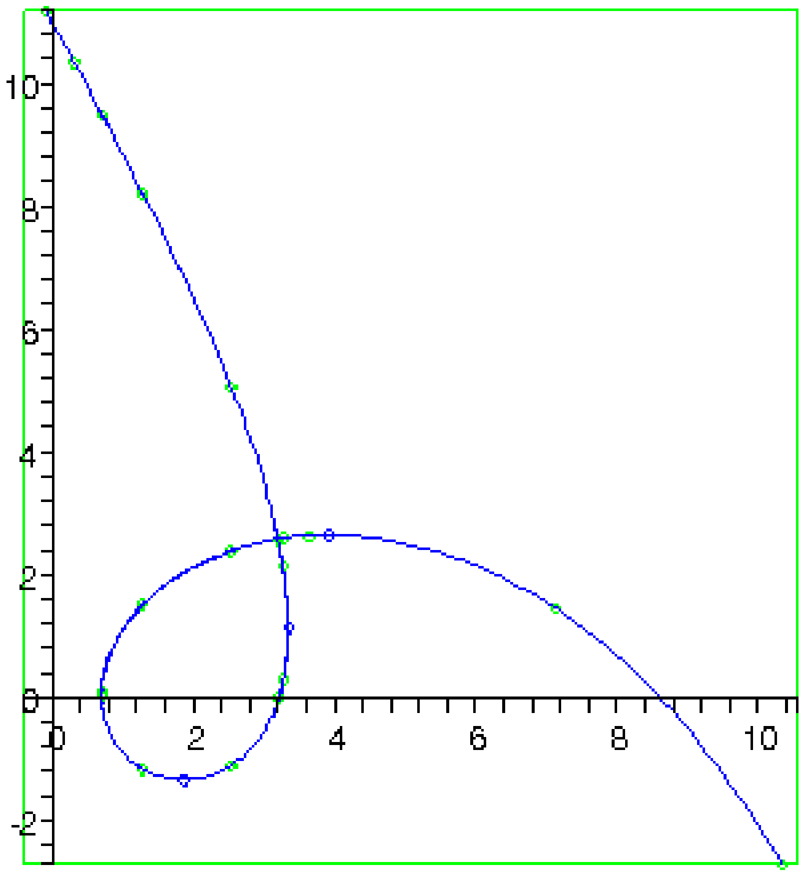
PPO

PCo

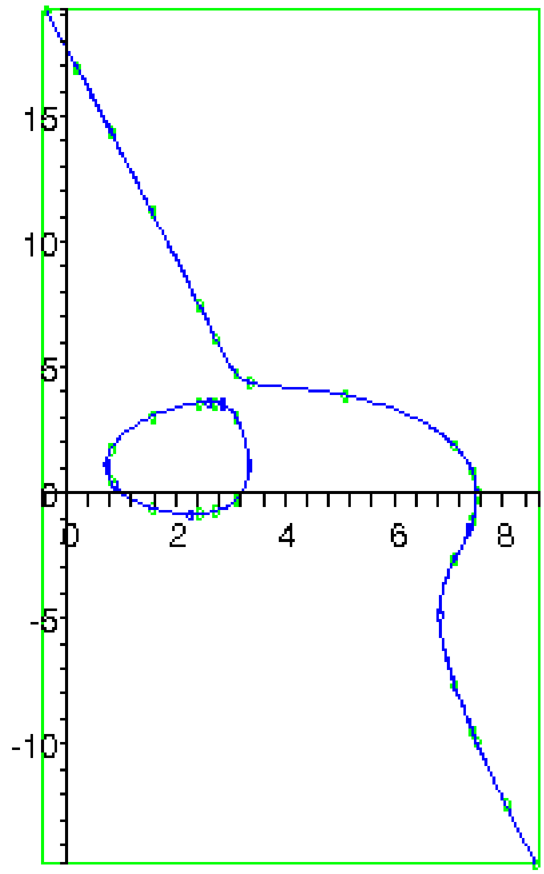



```
Warning, the name homology has been redefined
```

$cucusta\_p := -2744.036426\, x - 82.20445111\, y + 1469.657450 - 43.14610430\, x^2\, y + 73.19579374\, x\, y - 76.83289090\, x\, y^2 + 468.9821575\, y^2 + 960.1965790\, x^2 - 76.83289090\, x^3 - 43.14610430\, y^3$



$$cucu\_2D\_p := 44663.54144\,x + 14232.74446\,y + 322.8793638\,x^2 y - 4288.875252\,x y + 1283.454499\,x y^2 - 6415.020747\,y^2 - 15025.72064\,x^2 + 1283.454499\,x^3 - 31156.99726 + 322.8793638\,y^3$$



CALCOLO DEL **GRAFICO DELLA CUBICA**DI CURVATURA STAZIONARIA

x_pol:= -0.07927971648

y_pol:= 0.4002713949

x_pol:= 3.117361201

y_pol:= 2.988860171

x_pol:= 8.855012677

y_pol:= -8.792399670

x_pol:= 12.05165359

y_pol:= -6.203810894

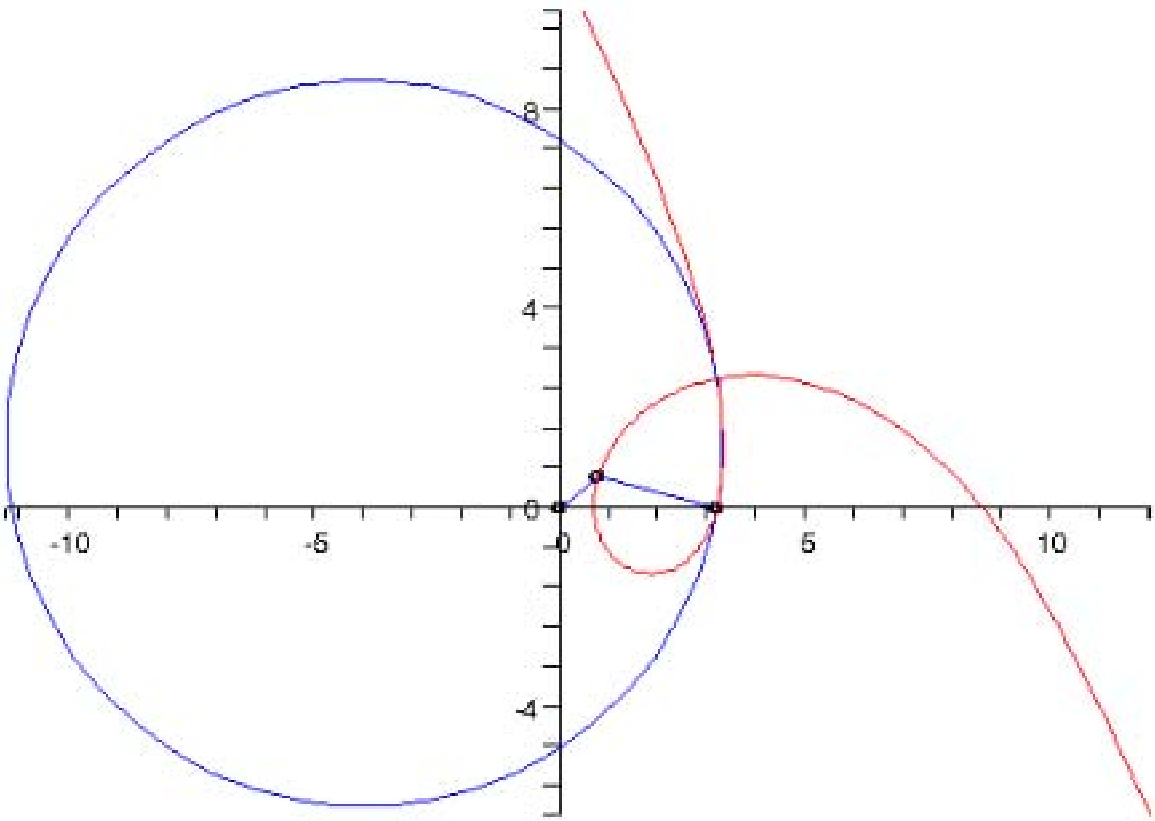


GRAFICO DELLA **DERIVATA**DELLA CUBICA DI CURVATURA STAZIONARIA

$$h21 := -\frac{1}{-2.179163359\ 10^{9} + 2.189383205\ 10^{9}\cos(\zeta)^{2}}\Big(0.1000000000\Big(-8.631222201\ 10^{9}\sin(\zeta)$$

$$- 4.075294972\ 10^{10}\cos(\zeta) + 4.039091052\ 10^{10}\cos(\zeta)^{3} + 5.94229271\ 10^{8}\sin(\zeta)\cos(\zeta)^{2}$$

$$+ 1.00000\ 10^{5}\,\mathrm{sqrt}\Big($$

$$-5.173721200\ 10^{11} \cos(\zeta)^2 + 1.012928555\ 10^{10} \cos(\zeta)^4 + 3.441486872\ 10^{11}$$
$$+ 1.631072544\ 10^{11} \cos(\zeta)^6 + 9.340741098\ 10^{10} \sin(\zeta)\cos(\zeta) - 9.773389875\ 10^{10} \sin(\zeta)\cos(\zeta)^3$$
$$+ 4.800292265\ 10^{9} \sin(\zeta)\cos(\zeta)^5 \Big) \Big) \Big)$$

$x\_pol := -0.04470106561$

$y\_pol := 0.2256889742$

$x\_pol := 3.151939851$

$y\_pol := 2.814277750$

$x\_pol := -.4198282404$

$y\_pol := 3.321751290$

$x\_pol := 2.776812677$

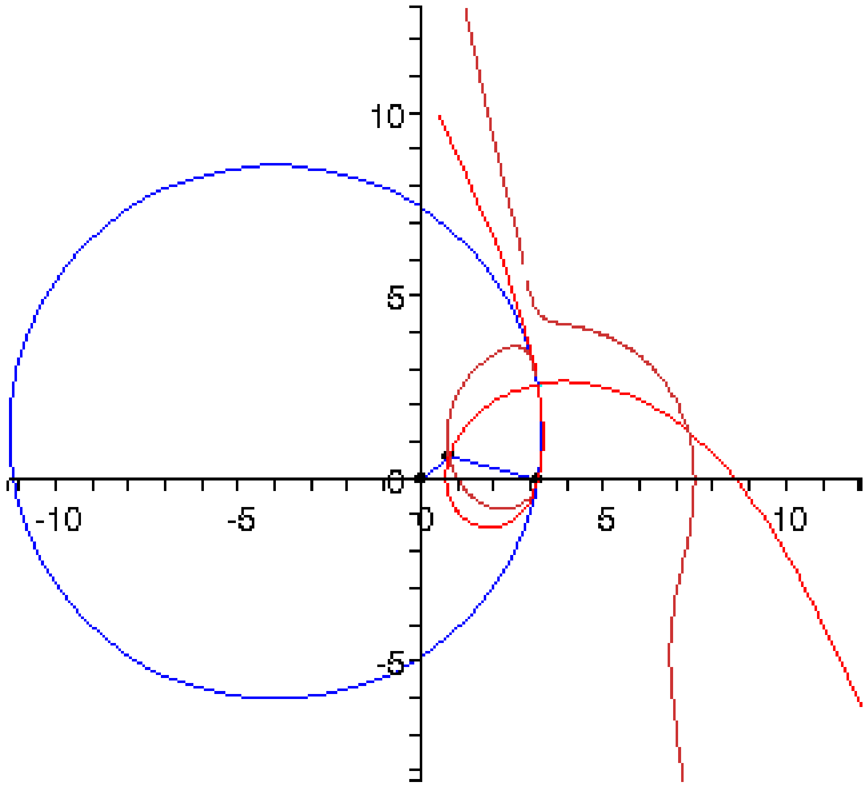
$y\_pol := 5.910340066$

$x\_pol := 4.002906680$

$y\_pol := -10.87765739$

$x\_pol := 7.199547597$

$y\_pol := -8.289068614$



$$6.213372139$$

$$\frac{1}{2.189383205\ 10^{19}\ \sin(\zeta)^2 - 1.021984591\ 10^{17}} \Big(0.5000000000 \Big(-1.607398586\ 10^{19}\ \sin(\zeta) - 8.078182104\ 10^{19}\ \sin(\zeta)^2 \cos(\zeta) - 1.188458542\ 10^{18}\ \sin(\zeta)^3 - 7.240784034\ 10^{17}\ \cos(\zeta) + \mathrm{sqrt}\Big(1.952109691\ 10^{38}\ \sin(\zeta)^2 + 3.525332568\ 10^{39}\ \sin(\zeta)^3 \cos(\zeta) + 1.356932409\ 10^{40}\ \sin(\zeta)^4 + 1.895217996\ 10^{37}\ \sin(\zeta) \cos(\zeta) + 6.525702611\ 10^{39}\ \sin(\zeta)^4 \cos(\zeta)^2 + 1.920116906\ 10^{38}\ \sin(\zeta)^5 \cos(\zeta) + 1.169847440\ 10^{38}\ \sin(\zeta)^2 \cos(\zeta)^2 + 1.412433707\ 10^{36}\ \sin(\zeta)^6 + 5.242895343\ 10^{35}\ \cos(\zeta)^2 \Big) \Big) \Big)$$

$$2.901702560$$

$$44663.54144\, x + 14232.74446\, y + 322.8793638\, x^2 y - 4288.875252\, x y + 1283.454499\, x y^2 - 6415.020747\, y^2 - 15025.72064\, x^2 + 1283.454499\, x^3 - 31156.99726 + 322.8793638\, y^3$$

$$-2744.036426\,x - 82.20445111\,y + 1469.657450 - 43.14610430\,x^2\,y + 73.19579374\,x\,y - 76.83289090\,x\,y^2 + 468.9821575\,y^2 + 960.1965790\,x^2 - 76.83289090\,x^3 - 43.14610430\,y^3$$

$$83.26076534\,\eta^3 + 83.26076534\,\eta\,\xi^2 - 28.85350106\,\eta^2\,\xi - 28.85350106\,\xi^3 + 640.6316838\,\eta\,\xi = 0$$

$$-1045.929452\,\eta^2 - 4866.805268\,\eta\,\xi - 1320.352486\,\eta\,\xi^2 + 90.4205548\,\eta^2\,\xi + 90.4205548\,\xi^3 - 1320.352486\,\eta^3 - 640.6316838\,\xi^2 + 9361.641225\,\eta = 0$$

$Burmester := \{\xi = 0., \eta = 0.\}, \{\xi = -1.982185386, \eta = -3.828475683\}, \{\eta = 0.1665801693, \xi = -1.688561858\}, \{\eta = 0.4582701967, \xi = -2.547080872\}, \{\xi = -1.499482648, \eta = 2.728197574\}$

$$soluz\_0 := \{\xi = 0., \eta = 0.\}$$

$$soluz\_0 := \{\xi 0 = 0., \eta 0 = 0.\}$$

$$BUR0x := 0.$$

$$BUR0y := 0.$$

$$soluz\_1 := \{\xi = -1.982185386, \eta = -3.828475683\}$$

$$soluz\_1 := \{\eta 1 = -3.828475683, \xi 1 = -1.982185386\}$$

$$BUR1x := -1.982185386$$

$$BUR1y := -3.828475683$$

$$soluz\_2 := \{\eta = 0.1665801693, \xi = -1.688561858\}$$

$$soluz\_2 := \{\xi 2 = -1.688561858, \eta 2 = 0.1665801693\}$$

$$BUR2x := -1.688561858$$

$$BUR2y := 0.1665801693$$

$$soluz\_3 := \{\eta = 0.4582701967, \xi = -2.547080872\}$$

$$soluz\_3 := \{\eta 3 = 0.4582701967, \xi 3 = -2.547080872\}$$

$$BUR3x := -2.547080872$$

$$BUR3y := 0.4582701967$$

$$soluz\_4 := \{\xi = -1.499482648, \eta = 2.728197574\}$$

$$soluz\_4 := \{\eta 4 = 2.728197574, \xi 4 = -1.499482648\}$$

$$BUR4x := -1.499482648$$

$$BUR4y := 2.728197574$$

$$BUR0xx := 3.196640917$$

$$BUR0yy := 2.588588776$$

$$BUR1xx := 7.315697607$$

$$BUR1yy := 1.315931380$$

$$BUR2xx := 3.331808944$$

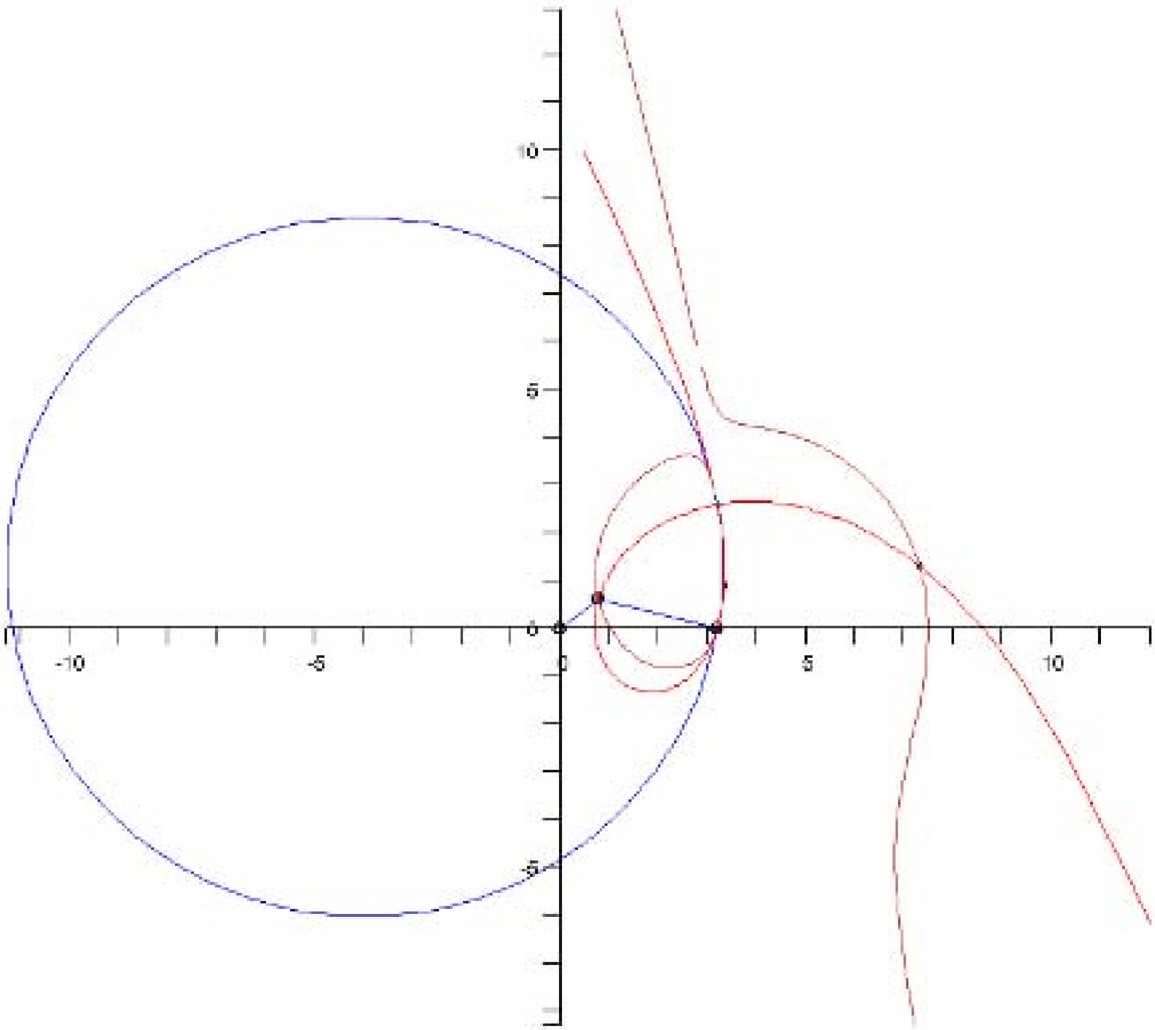
$$BUR2yy := 0.897222557$$

$$BUR3xx := 3.196810917$$

$BUR3yy := 0.000610323$

$BUR4xx := 0.777208650$

$BUR4yy := 0.629543685$



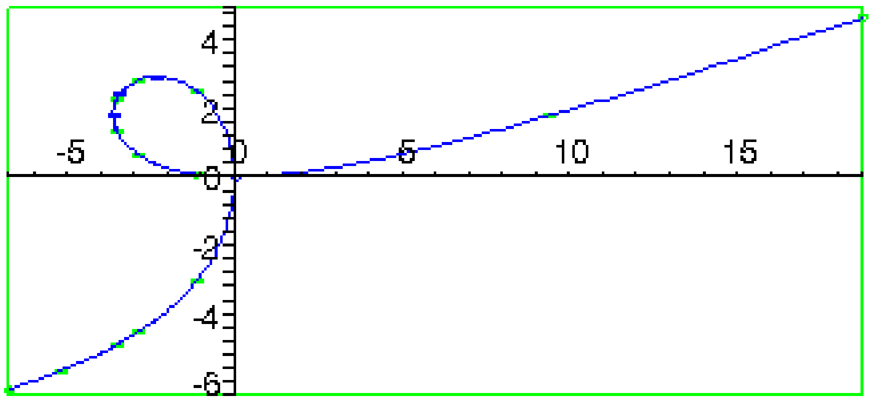
$cuxi := 83.26076534\, y^3 + 83.26076534\, x^2\, y - 28.85350106\, x\, y^2 - 28.85350106\, x^3 + 640.6316838\, x\, y = 0$

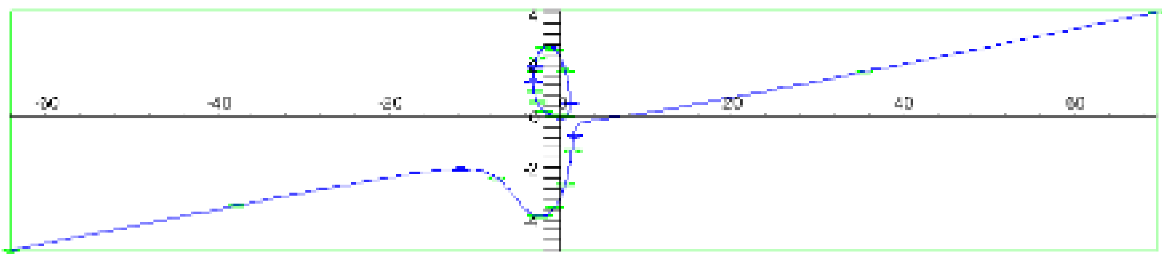
$cuxi2 := -1045.929452\, y^2 - 4866.805268\, x\, y - 1320.352486\, x^2\, y + 90.4205548\, x\, y^2 + 90.4205548\, x^3 - 1320.352486\, y^3 - 640.6316838\, x^2 + 9361.641225\, y = 0$

$cuxi\_func := 83.26076534\, y^3 + 83.26076534\, x^2\, y - 28.85350106\, x\, y^2 - 28.85350106\, x^3 + 640.6316838\, x\, y$

$cuxi2\_func := -1045.929452\, y^2 - 4866.805268\, x\, y - 1320.352486\, x^2\, y + 90.4205548\, x\, y^2 + 90.4205548\, x^3$

$$- 1320.352486\, y^3 - 640.6316838\, x^2 + 9361.641225\, y$$





$$ccs\_T := -\eta^3 b3 - \eta \xi^2 b3 - \xi \eta^2 a3 - \xi^3 a3 - 3 \eta^2 \xi b2 + 3 \eta \xi b2^2 - 3 \xi^3 b2$$

$$cdcs\_T := -4 \eta^2 b2\, a3 + 4 \eta \xi b2\, b3 + 5 \eta \xi^2 b2 - 4 \eta^2 \xi b3 - \xi^2 \eta b4 + 4 \xi^2 \eta a3 - \eta^2 \xi a4 - 4 \xi^3 b3 - \eta^3 b4 - \xi^3 a4 + 5 \eta^3 b2 + 4 \eta^3 a3 - 3 \xi^2 b2^2 + 3 \eta b2^3 - 9 \eta^2 b2^2$$

$$\xi := T \eta$$

$$-\eta^3 b3 - \eta^3 T^2 b3 - T \eta^3 a3 - T^3 \eta^3 a3 - 3 \eta^3 T b2 + 3 \eta^2 T b2^2 - 3 T^3 \eta^3 b2$$

$$-4 \eta^2 b2\, a3 + 4 \eta^2 T b2\, b3 + 5 \eta^3 T^2 b2 - 4 \eta^3 T b3 - T^2 \eta^3 b4 + 4 T^2 \eta^3 a3 - \eta^3 T a4 - 4 T^3 \eta^3 b3 - \eta^3 b4 - T^3 \eta^3 a4 + 5 \eta^3 b2 + 4 \eta^3 a3 - 3 T^2 \eta^2 b2^2 + 3 \eta b2^3 - 9 \eta^2 b2^2$$

$$ccs\_T := \frac{-\eta^3 b3 - \eta^3 T^2 b3 - T \eta^3 a3 - T^3 \eta^3 a3 - 3 \eta^3 T b2 + 3 \eta^2 T b2^2 - 3 T^3 \eta^3 b2}{\eta^2}$$

$$\eta := \frac{3 T b2^2}{b3 + T^2 b3 + T a3 + T^3 a3 + 3 T b2 + 3 T^3 b2}$$

$$equaz\_4 = \frac{1}{(T a3 + 3 T b2 + b3)^3 (1 + T^2)^2} (9 T b2^5 (T^4 a3^2 + 3 T^4 b2\, a3 - 3 T^3 b2\, a4 + 3 T^3 b2\, b3 + 6 T^3 a3\, b3 - 3 T^2 b2\, b4 - 3 T^2 b2^2 + 5 T^2 b3^2 - 3 T^2 a3^2 - 3 T^2 b2\, a3 - 2 T a3\, b3 - 3 b3\, T b2 + b3^2))$$

$$equaz\_4 = T^4\, a3^2 + 3\, T^4\, b2\, a3 - 3\, T^3\, b2\, a4 + 3\, T^3\, b2\, b3 + 6\, T^3\, a3\, b3 - 3\, T^2\, b2\, b4 - 3\, T^2\, b2^2 + 5\, T^2\, b3^2 - 3\, T^2\, a3^2 - 3\, T^2\, b2\, a3 - 2\, T\, a3\, b3 - 3\, b3\, T\, b2 + b3^2$$

$$\mathrm{RootOf}\big((a3^2 + 3\, b2\, a3)\, \_Z^4 + (-3\, b2\, a4 + 3\, b3\, b2 + 6\, a3\, b3)\, \_Z^3 + (-3\, b2\, b4 - 3\, b2^2 + 5\, b3^2 - 3\, a3^2 - 3\, b2\, a3)\, \_Z^2 + (-2\, a3\, b3 - 3\, b3\, b2)\, \_Z + b3^2 \big)$$